Strongly compressible current sheets under gravitation

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Abstract

Many stormy events in astrophysics occur due to the sudden magnetic energy release. This is possible if a magnetic configuration abruptly changes its topology, an event usually referred to as magnetic reconnection. It is known that pure Ohmic decay is inefficient, occurring during cosmological times (due to the huge characteristic scales L). It is recognized that the presence of current sheets speeds up the process, but still insufficiently[1,2,3,4,5]. We show that, in highly compressible and substantially gravitational media, the reconnection is fast enough to account for stormy events. Thus, highly compressible situations offer exiting opportunities in explanations of violent events, although full-scale compressible and gravitational simulations proved to be quite challenging.

Basically, there are two characteristic times: Huge Ohmic decay time, $t_o = L^2/\eta$, (η is resistivity), and sufficiently short Alfvén time,

$$t_A = \frac{L}{C_A},$$

C_A is Alfvén velocity, $C_A = B/\sqrt{4\pi\rho}$, the ratio of these times being the Lundquist number $S \gg 1$. The Alfvén time would be good enough to explain many stormy events in astrophysics. The problem is that the magnetic field would not change its topology that fast. In particular, the Sweet-Parker (SP) current sheet[1,2], with the width

$$\delta_{SP} = \frac{L}{S^{1/2}}.$$

results in shorter (than Ohmic) reconnection time, $t_{SP} = t_o/S^{1/2} = t_A S^{1/2}$. The (dimensionless) reconnection rate for SP is

$$R = \frac{t_A}{t_{SP}} = \frac{1}{S^{1/2}}. \tag{1}$$

For huge astrophysical values of $S = 10^{10\div 20}$, say, the SP mechanism is too slow. In other words, the exponent 1/2 in (1) is too big.

Much more efficient mechanism was suggested by Petschek[3], with reconnection rate,

$$R = \frac{t_A}{t_P} = \frac{\pi}{8\ln S},$$

which is sufficiently fast. It was pointed out however that this mechanism may only work under special conditions in the vicinity of the x-point, where the reconnection occurs[4,5,6,7], in particular, nonuniform resistivity[8,9]. The aim of this letter is to argue and present numerical evidence that strong compressibility and gravity result in such conditions, leading to Petschek-like reconnection.

We will deal with 2.5-dimensional case, when magnetic field is presented as $\mathbf{B} = \{\mathbf{B}_\perp, B_z\} = \{B_x(x,y), B_y(x,y), B_z(x,y)\}$. In the vicinity of the current sheet the configuration is in nearly 1-dimensional equilibrium, i.e.,

$$p + \frac{B_y^2 + B_z^2}{8\pi} = \text{const}, \tag{2}$$

where p is the pressure. In low pressure plasma (of stellar coronas), $\beta = p/(B^2/8\pi) \ll 1$, we can neglect the pressure in (2), leading to a field configuration depicted in Fig. 1: With

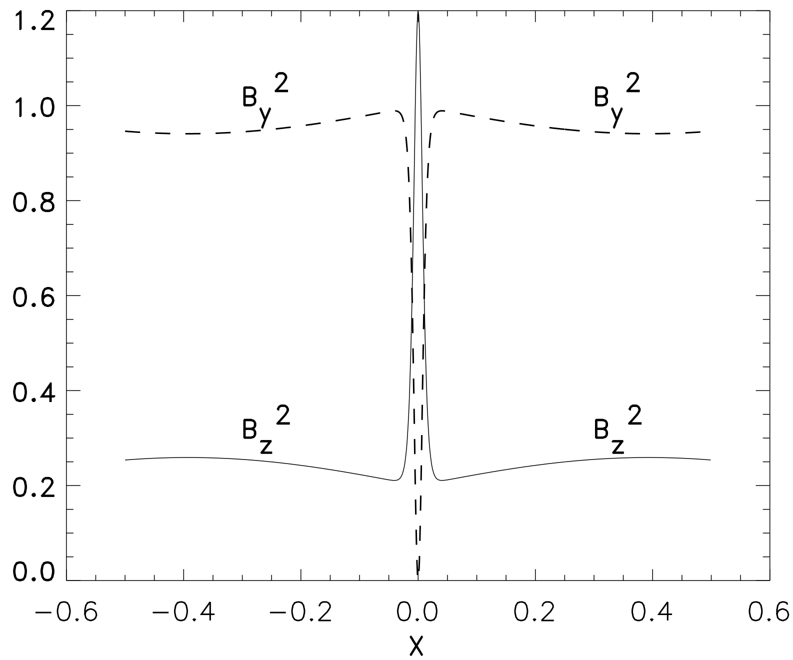

FIG. 1: Typical magnetic distribution in the vicinity of the current sheet. B_y-component abruptly changes sign at current sheet location $x = 0$, and therefore B_z-component acquires a sharp maximum.

sharp maximum of the B_z-component at $x = 0$. This makes $x = 0$ point singular[10] because this maximum will be "wiped out" by Ohmic dissipation. Indeed, it takes only $\delta_{PS}^2/\eta = t_A$, i.e., Alfvén time, for this maximum to disappear. As a result, the current sheet collapses, leading to density and pressure build-up, and speeding-up the reconnection rate. More detailed estimates[11] indeed show that, in the vicinity of the current sheet $\nabla \cdot \mathbf{v} < 0$, i.e., compression, and $\nabla \cdot \mathbf{v} \approx 1/t_A$. Thus, substantial compressibility creates a singularity – i.e., special conditions in the vicinity of the current sheet.

Still, the pressure build-up would try to stifle the collapse, and presumably slow-down the reconnection. We show that the gravity, present in astrophysical conditions creates additional special conditions and again facilitates the reconnection. If the scale height H is $< L$ (quite a modest requirement, satisfied in the Sun) the equilibrium condition (2) is incompatible with the gravity, unless the current sheet is strictly horizontal, which is unlikely

to happen. In the vicinity of the current sheet, the magnetic field is a function of x only, while gravity imposes $\sim \exp(-y/H)$-dependence for p, and (2) cannot be satisfied.

Thus, the reconnection starts to "feel" the gravitation when $L/H > 1$. In addition, we note, that dramatic build-up of density and therefore of pressure perturbations, is unlikely in the stellar atmospheres. Indeed, unless there are special conditions (satisfied, e.g., for the prominences), in the presence of gravity forces, the density "blobs" cannot be sustained in coronas for a long time, and they would slip down along the magnetic field lines.

To be more specific, for strongly gravitational stars, when more strict inequality, $L/H > \beta^{-1/2}$ is satisfied, or

$$Z = \frac{L\beta^{1/2}}{H} = \frac{Lg}{C_A^2\beta^{1/2}} > 1,$$

the fall down time is less than Alfvén time. We expect that, if $Z \gg 1$, then, roughly speaking, the matter remains in stratified and almost unperturbed state all the time during the reconnection process. This prevents the build-up of the pressure in the vicinity of the current sheet. Simple but more cumbersome estimates[11] confirm this statement.

Of course, the only reliable way to check these statements is to use numerical simulations. There are not so many compressible simulations although it is known that the compressibility would speed up reconnection process[12], and that was indeed observed experimentally[13]. The compressibility has a profound effect in the vicinity of magnetic null-points[14].

We provide "global" simulations, i.e., for the whole region. This makes it possibly to study the scaling exponents of the reconnection rate. We present three types of simulations:

1) $Z = 0$, no gravity, but substantial compression. A rosette-structure configuration was considered, in which case the current sheet formation is inevitable[15]. The plasma collapse was indeed persistently observed in simulations[16,11]. Thus, it was estimated that in order to reach equilibrium Fig. 1, B_z should increase 1.5 times, say, resulting in pressure increase only twice, while the observed pressure was increasing up to 30 times!

This process strongly depends on the compressibility: With increasing B_z-component, when plasma becomes nearly incompressible, old results about SP slow reconnection rate[4,5,6,8]. Note, however, that rosette-structure simulations cannot reach very high compressibility: If we take very small initial B_z-component, the magnetic configuration will

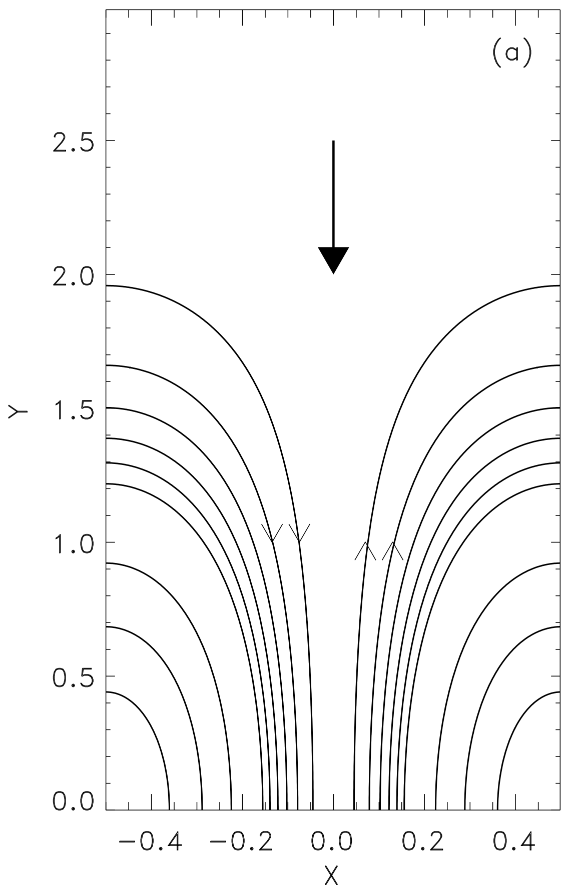

FIG. 2: Initial (a) and final (b) magnetic configuration in our simulations. Big vertical arrow represents the gravity force.

collapse in such a way that B_z is increased. Nevertheless, they definitely show that the current sheet collapse speeds-up the reconnection process: The reconnection rate scaling exponent is $= 0.27 \pm 0.02$, definitely smaller than 1/2, characteristic for the SP process.

2) $Z \gg 1$, extremely strong gravity. We can imagine that two pairs of "sunspots" of opposite polarities are initially far away from each other, and therefore they are not connected. Suppose that, in the course of evolution, they approach each other. Due to frozen-in conditions, the topology of the field lines is not easy to change, and, for a while, they remain disconnected from each other, Fig. 2(a). Due to finite conductivity, a current sheet will form, and the reconnection starts[17,18]. Further evolution is observed in our numerical simulations,

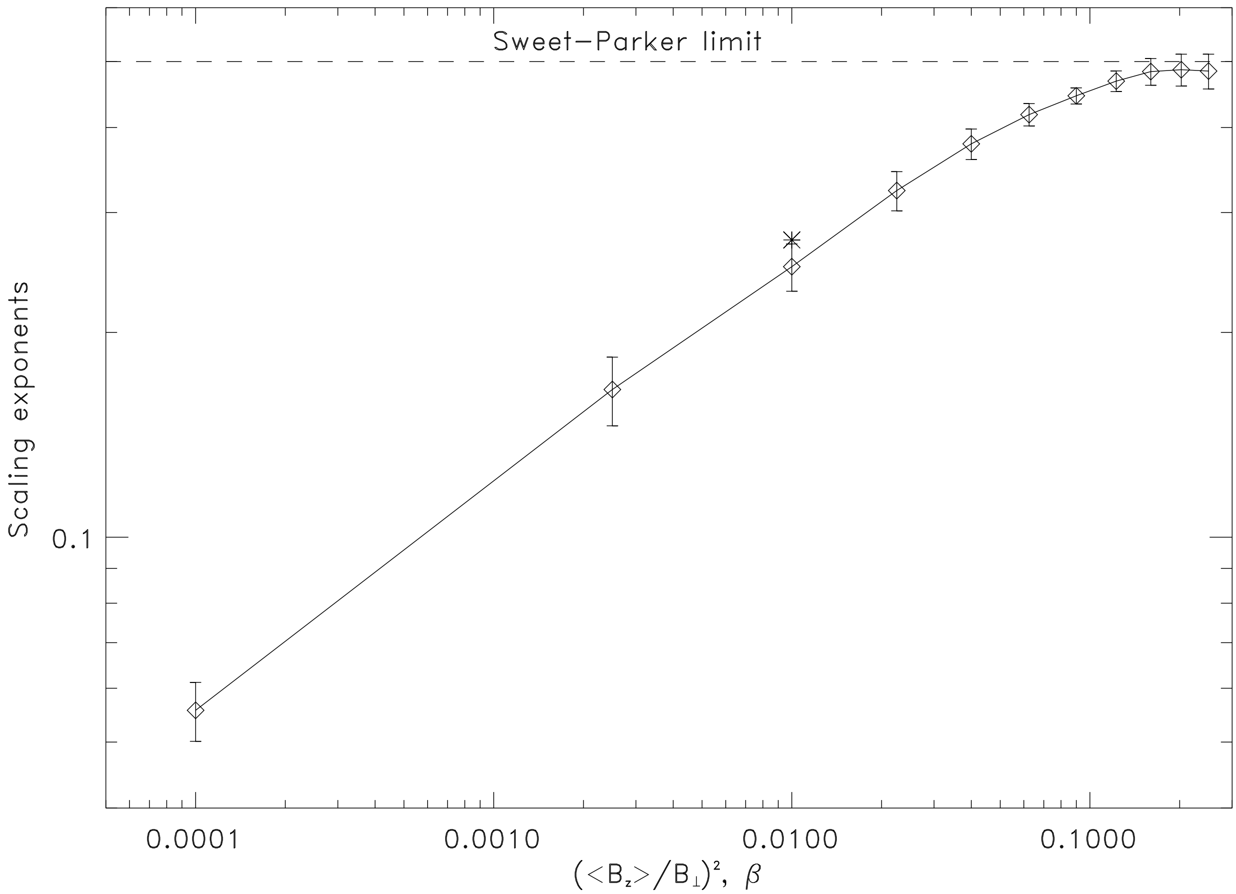

FIG. 3: The scaling exponents depending on compressibility. The diamonds correspond to the different values of $(B_z/B_\perp)^2$, and $Z \gg 1$, while the asterisk corresponds simulations with $\beta = 0.01$ and $Z = 0$.

with final (reconnected) state depicted in Fig. 2(b).

This time, the B_z-component can be arbitrary small, and therefore we can study extreme compressibility. We measured electric field (responsible for high energy particles acceleration) normalized on a maximal possible field, $\mathbf{v} \times \mathbf{B}$, where $v = C_A$, (which is essentially the reconnection rate, like (1)), for different Lundquist numbers S, to get the scaling exponents. We also provided simulations with different compressibility's, characterized by parameter $B_z/B_\perp$. When this parameter is small, the media is highly compressible, while increasing this parameter, we will approach essentially incompressible situation. As seen from Fig. 3,

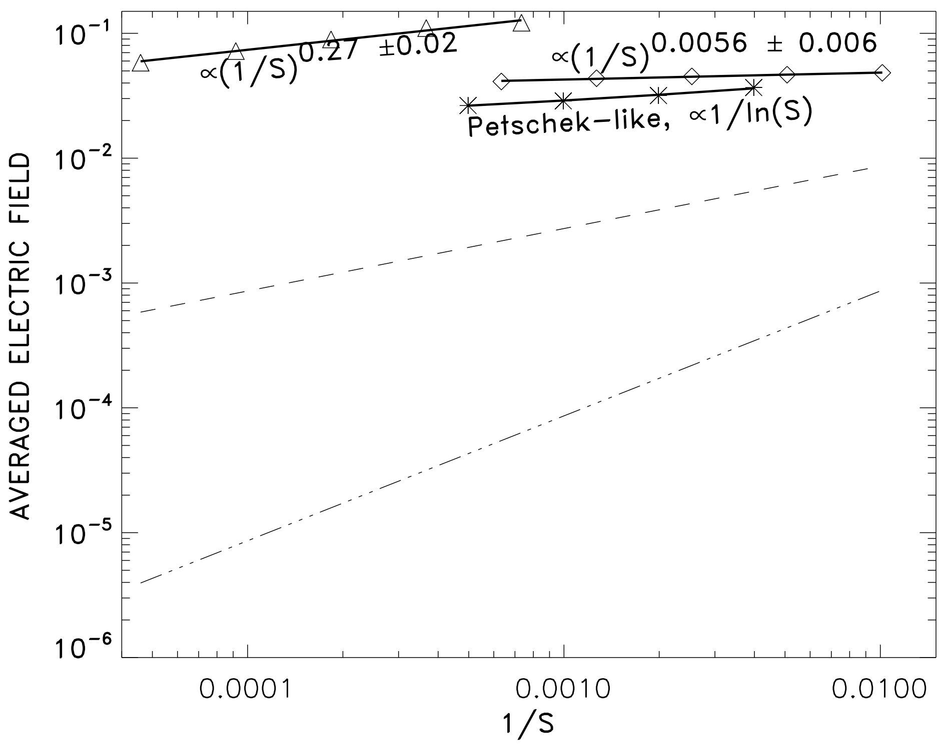

FIG. 4: Normalized averaged electric field measured in simulations. Triangles correspond to $Z = 0$ simulations of rosette-structure. Diamonds – to $Z \gg 1$, highly compressible case. Asterisks correspond to $Z = 0.2$, solar conditions, fitted with Petschek-like scaling. Short dashes correspond to Sweet-Parker, $\sim 1/S^{1/2}$, and the dashed-dotted line corresponds to pure Ohmic decay, $\sim 1/S$.

the incompressible media is reached when $(B_z/B_\perp)^2 = .04$, and then the Sweet-Parker reconnection is recovered. For higher compressibility's we found that the reconnection is much faster (i.e., the scaling exponents are mach smaller). The fastest reconnection proceeds essentially with Alfvén time. It is obvious that the compressibility speeds up the reconnection process in the presence of strong gravitation.

3) Full-scale calculations with arbitrary Z, and arbitrary compressibility's are not easy to perform. We managed to simulate conditions for the Sun: $L/H > 1$, so that special conditions do appear at the vicinity of the x-point due to the gravity, although $Z = 0.2 \div 0.3$. So far, we fail to simulate typical $\beta = 0.01$, or so, but we do have the case $\beta = 0.001$ (the field is 3 times stronger than typical). The results for all three types of simulations are

summarized in Fig. 4. For solar conditions, the scaling is close to Petschek-like rate, the standard deviation of measured electric field from Petschek-like being = 0.01. The simulations reveal rather stormy developments: Creation of a current sheet, and fast reconnection, see Supplement movie[19].

All these simulations show the importance of strong compressibility and gravity forces in explaining stormy and violent magnetic events in astrophysics.

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19. Watch the movie on the Web, http : //flash.uchicago.edu/˜samuel/Gravitational_Reconnection.